INSTITUTIONS

OF ASTRONOMICAL

CALCULATIONS;

CONTAINING A

SURVEY

OF THE

SOLAR SYSTEM,

WHEREIN

All its Dimensions of the Distances, Diameters,
Magnitudes and Velocities of the Sun, the
Earth, the Primary and Secondary Planets,
the Comets and the Stars are computed in
English Miles from the late discovered

PARALLAX of the SUN,

BY THE TWO LAST

TRANSITS of VENUS.

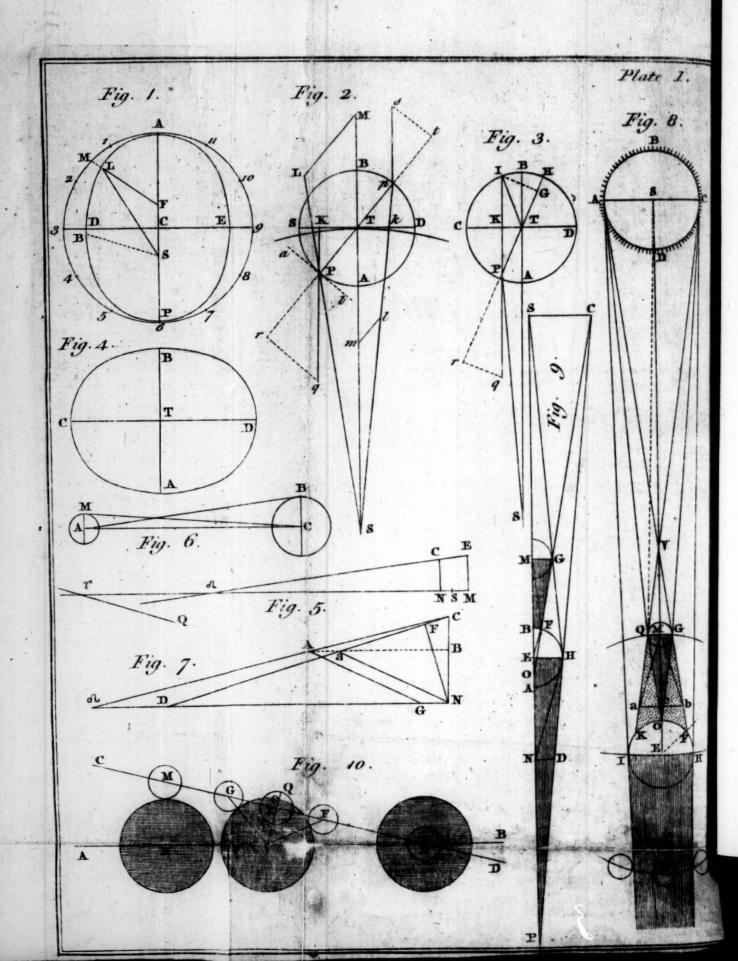
WITHA

Description of two New Pieces of Mechanism for exhibiting Artificial Views of such Transits.

Embellished with two COPPER-PLATES.

By BENJAMIN MARTIN.

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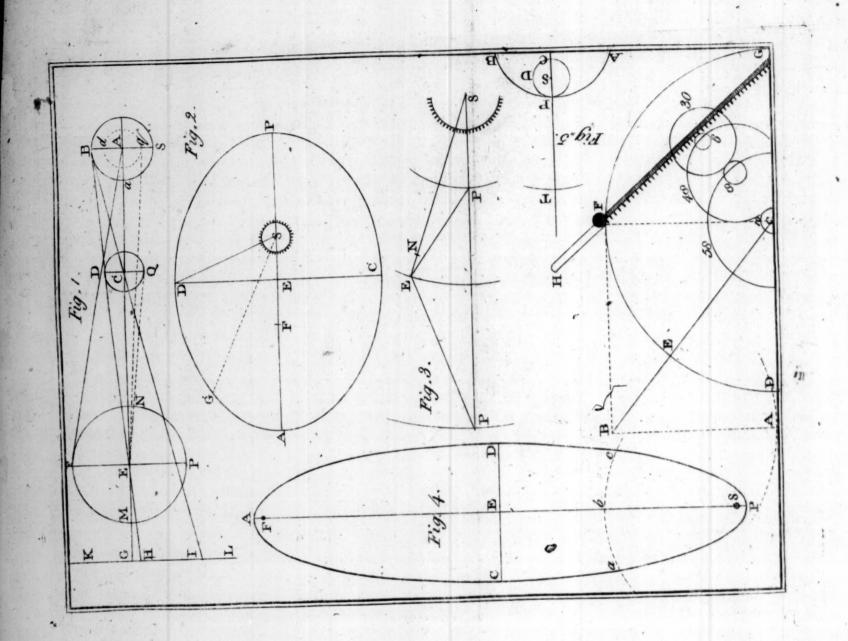
PREFACE.

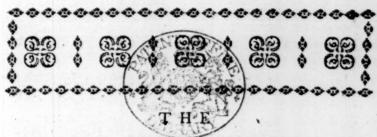
I Do myself the Honour of being the First to lay before the public Eye the most August of all Ideas, viz. that of A SURVEY of the SOLAR SYSTEM, expressing all its DIMENSIONS in our own MEASURE, of English Statute MILES. This I have prefumed to undertake for the following Reasons, viz. (1.) I have waited more than 12 Months, to give any abier Hand an Opportunity of affuming the same Task; but no one has appeared in so public and interesting a Cause. (2.) These Sorts of CALCULATIONS naturally follow those of the TRANSITS of VENUS, as they are the direct Consequence of a Discovery of the SUN's PARALLAX thereby. (3.) The NOVELTY of fuch a Work (as being the first from the CREA-TION,) joined with its supreme Usefulness, as the most curious and essential IMPROVEMENT of Astronomical Science, cannot fail of rendering it truly delectable to every Person of RELI-GION, TASTE, and LEARNING. (4.) Laftly, I could think of no better Way of employing my leisure Hours, than in surveying and illustrating the wonderful WORKS of creating Power and WISDOM, in the familiar Dimensions of all the magnificent PARTS and SCENERY of the Mundane System, as far as we are acquainted therewith.

The

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SOLAR SYSTEM furveyed;

AND THE

DISTANCES, DIAMETERS, MAGNI-TUDES, and VELOCITIES of the SUN, EARTH, PLANETS and COMETS computed, and now first stated in English Statute Miles.

N the Treatise on the Transit of I Venus, it has been shewn, that a just Estimation of the Dimensions of the Solar System is no other way to be obtained but by Observations on the Transits of Venus over the Solar Disk; and as these observations have been now lately made on two of those Transits, the agreement between them, has been found so great, that the Sun's Parallax is undoubtedly as nearly now determined as it can ever be expected, and thereby the Dimensions of the Planetary System, and of the Bodies which compose it, are very easily deducible on the Principles, and in the manner, following.

4122 The Observations made on the last Venereal

Transit, were at the 5 following places, viz.

1. At Wardhus (in Lapland) Latitude 70°. 22'. 36". N. and Longitude (from Greenwich) 31°: 5': 30": E. By Father Hell.

2. At Kola (in Lapland) Latitude 68°: 52': 56" N. and Longitude 33°: 2': 11" E. By Mr. Rumonsky.

3. At Hudson's Bay, Prince of Wales's Fort.
B Lat.

Latitude 58°: 47': 32". N. Longitude 94°: 12': 22": W. By Mcff. WALES and DYMOND.

4. At VILL St. Joseph, California; in Latitude 23°: 3': 37": N. Longitude 109°: 40'. 37": W. By Aebe Chappe.

5. At KING GEORGE'S ISLAND, in Royal BAY; Lat. 17°: 28': 55": S. Longitude 149°: 28': 24" W. By Meff. Capt. Cook, C. Green, and Dr. Solander.

4123. It has been shewn (Art. 4068) that the Time of the Whole DURATION of the TRANSIT as viewed from the Center of the Earth, was 6 Hours 20': 38"; that the time between the external and internal Contacts was 18': 36"; and therefore the Duration of the Transit between the two internal Contracts was 5h. 43': 26": And if it be supposed that the Solar Parallax on the day of the Transit was 8", 7, then by proceeding as directed (at 4077) the duration of the said Internal Transit will be found, for each of the places above-mentioned; and from the Differences between these, and the Observed Durations, the true or real Parallax of the Sun is inferred by the Rule in (4128, &c.)

4124. These observed Durations, with the Differences both of the computed and observed Durations from the Central One, and the Parallaxes belonging to each, are contained in the following Table.

	Obs. Durations.	Diff. of comp. Durations.	Diff. of Obf. Durat.	Sol. Parall
K. Geo. Ifland.	H. 5: 29: 52,5			"
Wardhus.	5:53:14	\$23:31,36 15:48,93	23:21,5 15:51,6 7:50,3	8,639 8,724 8,905
Kola.	5 : 53 : 19	\[\begin{cases} 23:41,09 \\ 16: 4,41 \end{cases} \]	23: 26,5 15: 56,6	8,611
a	5 : 45 : 23,7			8,511
California.	5: 37: 32,4	7:42,43	7: 29,9	8,464





4125. The Mean of all the above Parallaxes is 8,65; this is so near to 8,692", the Parallax of the Sun determined from the Observations of Mess. Mason and Dixon, at the Cape of Good Hope on the former Transit of 1761, and a Comparison thereof with many others, that there is now no hopes of any greater Accuracy in this important affair, at least for 600 Years to come, as appears for (4013) And therefore we shall compute from this Parallax, what are the Dimensions of the Solar System.

the Doctrine of Parallaxes, it will be necessary to premise a sew Definitions and Explications on that Head, for the Sake of such Readers as may not have seen the abovementioned TREATISE on the TRANSITS of VENUS. Therefore (in Fig. 1.) let A be the Center of the EARTH, C that of the PLANET Venus, and E that of the Sun; all joined by the right Line AG, terminated in the starry Firmament KL. Also let AB be a Semidiameter of the Earth at right Angles to AG, and draw the Line BCI.

4127. Then it is evident, an Eye placed in the Center of the Earth at A, or on its Surface at a, views the Center C of Venus in the Firmament at G by the vifual Ray AG: But an Eye at B (90° distant from a) will view the same Center C, in the Heavens at I, by the Ray BCI; and this Difference GI of the apparent Place of the Planet C, is called the PARALLAX of that PLANET; and is always measured by the Angle GCI, or its equal ACB, which is therefore called the Parallatic Angle, or simply, the PARALLAX of the Planet, Venus for Instance.

4128. Hence it appears that when any Planet C appears in the Horizon AC to a Spectator at B, its Parallax is then called the Horizontal Parallax, and is B 2

that Angle under which the perpendicular Semidiameter AB of the Earth appears at the Planet C.

4129. After the same manner it is shewn that an Eye at A would view the Center E of the Sun in the Heavens at G; but a Spectator at B would see it at H, and the Difference of these apparent Places GH is measured by the Angle GEH, or AEB, and is therefore called the SUN'S Horizontal PARALLAX, the Quantity of which is

now determined to be at a mean, 8,78.

4130. Lastly; a Person at B will see the sun at H and the Planet at I; and therefore the Difference HI, is called the Parallax of the Planet from the Sun, and is measured by the Angle IBH.

others must be added relative to the Orbits, and Distances of the Planets from the Central Sun. The Orbit of a Planet is not Circular but Eliptical, such as ACPD (Fig. 2.) whose Center is E; AP is the Axis, CD the Diameter; F and S the two Focuses of the Ellipse, in one of which the Sun S always resides.

4132. Hence it is evident that a Planet revolving in fuch an Orbit about the Sun, will be always altering its Distance from the Sun; for at A it will be farthest of all from the Sun S; therefore the Point A is called the Aphelion, and AS the Aphelion Distance. Again, when the Planet is at B, it is nearest of all to the Sun; for that reason the point B is called the Perihelion. Lastly when the Planet is at C or D it is then at a Mean Distance SD from the Sun, because from the nature of the Ellipse, the Line SD is equal to EP or AE, and is therefore an arithmetic Mean between the Greatest and Least Distances AS and SP.

4133. In Confequence of the Planets unequal Distance from the Sun, the apparent Diameter of the Sun will be always variable inversely with the Distance, that is, it will

be least at A, greatest at P, and a Mean at D. And in like manner, the Angle which the Diameter of the Planet subtends at the Sun (which is double it's Parallax) will be ever variable in the same Ratio.

4134. And then, with respect to the EARTH, the Line AP cuts the Ecliptic in 8°: 42': 52" of Cancer and Capricorn (see 3680) therefore it is farthest from the Sun at A, on the 29th of June; and nearest to it at P, on the 30th of December. But on the Day of the last Transit of Venus, the place of the Sun was 13°: 21 of Gemini, 25°: 22', from the Aphelion A, viz. at the Point G.

4135. Consequently the Solar Parallax at G (viz. 8,65") is less, than the Mean Parallax at D, in the Ratio of SD to SG; that is, by the Theory of the Earth, it is SD: SG:: 8,65: 8,78, the Parallax of the Sun when the Earth is at D or C.

4136. We are now prepared to compute the MAGNITUDE of the SUN and PLANETS, and the DIMENSIONS of their feveral ORBITS, from the known Diameter of the Earth, the Sun's Parallax, and the proportional mean Distances and Diameters of the several Planets. The mean Distance SD of the Earth being divided into 100000 equal Parts, the others are stated by Sir I. NEWTON (from their periodical Times) as follows:

Mercury. Venus. Earth. Mars. Jupiter. Saturn. 38710. 72333. 100000. 152369. 520096. 954006.

4137. With regard to the Diameter of the Earth, that is also variable from the Equator to the Pole, as the Earth is not a SPHERE, but a SPHEROID; having the longest Diameter through the Equator 7970,8 English Miles; and its shortest from Pole to Pole 7932 (see MARINER'S MIRROR, Page 17.) The Mean of these is 7951,4, which must be taken for the Diameter of a Globe just equal to the solid Content of the Earth.

4138. The

4138. The apparent mean Diameter of the Sun is (by MAYER'S Tables) 32': 06"=1926", and the apparent mean Diameter of the Earth at the Diffance of the Sun is 17,56, being double the Parallax (4135) But the real Diameters are proportional to the apparent ones; and therefore, As 1926": 17,56:: 10000: 109,67= proportional Diameter of the Earth.

4139. The proportional Diameters of the Planets are collected from measuring their apparent ones by a Micrometer, and other aftronomical Methods, as follows: Let S be the Sun (Fig. 3.) E the Earth, and P a Planet; and let SN be the mean Distance of the Earth from the Sun. Then E and P being the Places of the Earth and Planet, at the Time when the apparent Diameter of the Planet is measured, there will be known, from the aftronomical Tables, the Sides SP and SE, in the Triangle SPE; and also the Angle ESP, and consequently the Side EP will be known.

4140. Then it will be as the Distance SP is to the Distance EP, so is the apparent Diameter of the Planet seen from E to its apparent Diameter seen from S. But the apparent Diameter of the Planet P seen from S is to the same as it would appear at the mean Distance of the Earth in N, as SN to SP.

with a MICROMETER, adapted to the Hugenian TE-LESCOPE of 123 Feet Object-Glass; from which Sir I. Newton infers, that this Planet, at his mean Distance from the Sun, would subtend an Angle of 16"; and therefore if Saturn were viewed at the mean Distance of the Earth from the Sun, he would subtend an

Angle of 152,641; because, 100000: 954006::16":
152,64096. Hence, the Sun's real Diameter is to Saturn's as 1926": 152,641::10000: 792,53.

Diameter with the same Micrometer and Telescope, and from thence it was computed by Sir Isaac, that at his mean Distance from the Sun, he would subtend an Angle of 37", and consequently if seen at the mean Distance of the Earth from the Sun, the Angle subtended would be 192,417. Therefore, the Sun's Diameter is to that of Jupiter, as 1926": 192,417:: 10000: 999,04.

found his apparent Diameter did not exceed 30", when his Distance was to the mean Distance of the Earth from the Sun as 15 to 41; and consequently if we say, As 41: 15:: 30": 10,9756; then this last Number will be that which we should use to find the proportional Diameter of Mars; but some modern Astronomers have thought the above apparent Diameter too large, tho' the ACADEMY of SCIENCES at PARIS have stated it at 11,4; we shall therefore take, as near the Truth, 10,5; and then we have this Analogy, As 1926": 10,5:: 100000:

54,517.

4144. As to the Planet VENUS, we have had two of her TRANSITS over the SOLAR DISK lately, which have afforded Means to measure her apparent Diameter at the Distance of the Sun, much more exactly than it could ever be obtained before. In the Transit of 1761, it was concluded from various Measurements to be about 58"; but in the last Transit, 1769, it was concluded from many Measures taken with different Micrometers to be between 57" and 58", the American Astronomers state it at 57,12; we shall therefore suppose 57,5 not wide from Truth; and therefore, as 100000: 27667: 57,5: 15,908 the Angle she would subtend at the mean Distance

Distance of the Earth from the Sun. So the Analogy is, 1926": 15,908:: 10000: 82,599.

4145. The apparent Diameter of the Planet Mercury seems never to have been accurately measured till the last Transit, November 9, 1769. For the Royal ACADEMY at PARIS stated it at 6,7" so lately as the Year 1767. But the Gentlemen of the American Philosophical Scriety, by a Mean of many repeated Measures of this Planet (with the best Sort of Micrometer, and with the utmost Attention) whilst it was traversing the Solar Disk, sound it to be 8,22" as viewed from the Earth; therefore, if viewed from the Sun at the Distance of the Earth, it would subtend 5,038; for 100000: 61290: 8,22: 5,038. Then its proportional Diameter is had by saying, as 1926": 5,038: 10000: 26,158.

4146. Thus we have found the Proportional Diameters of all the Planets, compared with that of the Sun, to be as follow, viz.

Sun. Mercury. Venus. Earth. Mars. Jupiter. Saturn. 10000. 26,158. 82,599. 109,67. 54,517. 999,04. 792,53. From hence the real and comparative MAGNITUDES of the Sun and Planets become known, because all Spheres or Solids are proportional to the Cubes of their Diameters, or like Sides. (Art. 675 and 841.)

4147. There remains now only one Thing to compleat the Dimensions of the Planetary System, and that is to measure the Velocity of the Planets Motions, or to find and express the Spaces in English Miles, which they describe in a given Time, at their mean Distances from the Sun, in their several Orbits respectively.

4148. But previous to this, and all the future Calculations of the same Kind, the Distance of the Earth from the Sun AE (Fig. 1.) must first be sound in Miles of our

own Measure, for which, in the right-angled Triangle AEB, there is known the Side AB=3975,7 (4137) and the Angle AEB=8,78 to find the Side AE, by the following Analogy;

As the Tangent of AEB=8,78—5,629069

Is to Radius - - - 10,
So is the Side AB=3975,7 — 3,599414.

To the Side AE=93399590 — 7,970345

Such is the Number of English Miles in the mean Distance of the Earth from the Sun SD (Fig. 2.) and thence it follows (since SD=EP), that twice that Distance is equal to the Transverse or longest Axis AP of the Earth's Orbit. In MAYER's Solar Table VII. (Page 13.) we find the greatest Distance AS is to the least SP, as 101680 to 98320; and consequently their Difference 3360 will be equal to FS, the Half of which ES=1680. Hence SD: SE:: 100000: 1680:: 93399590: 1569108. Then SD-SE=SP=91830482 English Miles for the least Distance of the Earth from the Sun: and SE+SD=94968698 Miles for the Earth's greatest Distance SA; the Difference of which is 3138216=SF.

4149. The mean Distance of the Earth being thus found in English Miles, those of the other Planets are immediately known from thence. Thus for MERCURY the Analogy is, as 100000: 38710:: 93399590: 36155000, which is his mean Distance from the Sun in Miles of our Measure; in this Manner the mean Distances of the other Planets from the Sun are found, as expressed in the general Table hereaster (4154.)

4150. In the fame Manner the Diameter of each Planet known (from their proportional Diameters (4146) in English Measure. Thus for MERCURY and the EARTH

you say, as 109,67: 26,158:: 7951,4 Miles: 1896,5 Miles, the Diameter of that small Planet. So the Diameters of all the other Planets, and the Sun, are deduced, as in the said Table you see them expressed.

will be as the Cubes of their Diameters; and therefore if we suppose the Magnitude of Earth to be divided into 10000 equal Parts, the Magnitudes of the Planets may be compared therewith, and expressed in the same equal Parts. Thus we say, as the Cube of 109,67: Cube of 26,158: 10000: 135,69, the Magnitude of the Planet Mercury; and therefore the Bulk of the Earth is to that of Mercury, as 10000 to 135,69; or as 73,7 to 1, very nearly. So for the rest as in the Table.

4152. In the last Place, for the VELOCITY of the PLANETS in their Orbits at their mean Distances, it is had by observing what their mean MOTIONS are in the astronomical Tables in one Minute, which must be expressed in Seconds of a Degree. These are stated by Dr. HALLEY, as follows.

Mercury. Venus. Earth. Mars. Jupiter. Saturn.
" 10,233. 4,000. 2,467. 1,317. 0,200. 0,083.

4153. Now from the Principles of Plain Trigonometry, it evident that as Radius is to the Tangent or Arch of 1", so is the mean Distance of the Planet from the Sun in English Miles, to the Length of that Arch in the same Measure. Thus, with respect to Mercury, we say,

As the tabular Radius - - - 10,0000000

Is to the Tangent of 1" - - - - 4,685575

So is \$\tilde{g}\$'s Diffance in Miles 36155000 7,558168

To the Length of the Arch of 1" 175,285 2,243743

Which multiplied by the Motion in one Minute - \} 10,233 1,010003

Gives the Velocity in 1', viz. 1793,7 - 3,253746

the

4154. So that the Planet MERCURY, at his mean Distance, moves at the Rate of 1793,7 Miles per Minute; and in this Manner the Velocity is found for the other Planets, as in the Table following, which contains the DIMENSIONS of the Solar System of Primary Planets in one general Synoptic VIEW.

	Distan-		Diameters in English Miles.	Propor- tional Magn.	ty per Mi-	Periodical Times.
Mercury	3871c	36155000	1896,5	135,69	M.les.	D. H. 1. 11 87. 23. 14. 34
Venus Earth	73333		5988,7	4272,3		224. 16. 41. 30 365. 05. 49. 25
Mars	152369	142312000	7951,4	1228,4		686. 22. 18. 10
		485767060		7559415		10750. 13. 14. 42

4155. The Diameter of the Sun in English miles is found by faying, as the Earth's proportional Diameter 109,67 is to its real Diameter 7951,4 Miles, so is the Sun's proportional Diameter 10000 to 725030 English Miles in his real Diameter.

4156. By the Motion of the Spots upon the Disk of the Sun, it is found that he has a Motion of Rotation about his Axis, which is also observed to be inclined to the Plane of the Ecliptic in an Angle of about 7°. 30'. And surther, that this Motion of the Sun about its Axis, is made in 25 Days, 15 Hours, 16 Minutes, that is, in 36916 Minutes. Consequently, if we multiply 725030 by 3,14159, and divide that Product by 36916, we shall have for the Quotient 61,7 Miles per Minute for the Velocity of Motion in the Middle or Equatoreal Parts of the Sun's Body.

4157. The MAGNITUDE of the SUN with respect to our EARTH, is known by comparing the Cubes of the proportional Diameters; thus the Cube of 109,67 is to the Cube 10000, as I to 758140; and so many times

the Sun is bigger than the Earth, viz. Seven Hundred Fifty-eight Thousand One Hundred and Forty Times. In the same Manner his Magnitude is compared with that of any other Planet.

4158. By the Discovery of the Sun's Parallax, we are able to correct the very erroneous Notions, we before had imbibed concerning the Distance, Magnitude, Velocity, &c. of the Moon, which are now all ascertained as follows. For, it is evident, that, if A, C, E be the Centers of the Earth, Moon, and Sun (Fig. 1.) The mean Distance of the Moon AC is to that of the Sun AE as the mean Solar Parallax 8,78=AEB is to the mean Lunar Parallax 57. 45 = ACB, that is, as 8,78 to

to 3465" or as 1 to 394,6. (See Table X. at 4001.)

4159. Then if the Sun's Distance 93399590 Miles (4148) be divided by 394,6 the Quotient will be 236664. English Miles in the Distance AC from the Earth's Center.

4160. The Diameter of the Earth is to the Diameter of the Moon as the Horizontal Parallax ACB to the apparent Semidiameter CD of the Moon, which by the best Observations, are as 50 to 15; therefore as 50: 15:: 3975,7: 1192,7. Whence the Diameter of the Moon is 2385,4 English Miles.

4161. The mean Motion of the Moon per Minute is 33,45 by the celebrated CLAIRAUT's tables (P. 89) and by proceeding as at (4152), We shall find the Velocity or Space described by the moon at her mean Distance in one Minute is 38,38 Miles. Whence the Velocity of the moon is to that of the Earth as 38,38, to 1117,1 or as 1 to 29 nearly.

4162. As the Eccentricity of the Moon's Orbit is always variable from the least 4333 to the greatest 6677, of fuch parts

parts whereof 100000 make the Mean Distance, it will happen that the Moon's greatest and least Distances from the Earth will also be constantly variable within those Limits, the least of all being 93323; and the greatest 106677; these reduced to English Miles, are 220864 and 252469: So that the Moon is sometimes nearer to us by 31605 Miles than at others.

4163. The proportional Magnitude of the moon to the Earth is found from the Cubes of their Semidiameters thus, As 3975,7: 1192,7:: 10000: 270; that is, the Magnitude of the Earth is to that of the Moon as 37 to 1 nearly.

much less than that of the Earth, (4161) is the Reason why she can never be seen retrograde from the Sun in her lower Conjunctions with the Earth, or when she is new, and passes between the Sun and the Earth; for in that Case her apparent regressive Motion being less than her real progressive Motion derived from the Earth, she must even then be direct in Motion.

Miles, his Semidiameter is 36217 Miles; and fince it is by the Micrometer the Distance of his 4 SATELLITES or Moons from his Center are measured in Semidiameters of his Body; and those Distances are known to be 5,67—9—14,38—25,3; therefore if by each of these Numbers you multiply 36217, the Products will be the Distances of the Satellites respectively in English Miles, as in the subsequent table.

4166. The mean Motions per Minute of these Moons are in Dr. HALLEY'S Tables as sollow; the first 508,98; the Second 253, 02; the Third 126"; and the sourth

54": from whence by the Rule at (4153) you find their respective VELOCITIES in Miles, as in the Table. The Velocity of the first Satellite is greater than that of its Primary, and therefore about its lower Conjunction, will appear from the Sun retrograde; but the other 3 will always be direct, as their Velocities are less than Jupiter's (4154.)

		Distances in Eng. Miles.	Periods of Revo- lution.	Velocity per Min.
First Second Third Fourth	9,00	205350,4 325953 520800	D. H. " 1. 18. 27. 34 3. 13. 13. 42 7. 3. 42. 36 16. 16. 32. 9	506,73 399,84 318,14

4167. The 5 SATELLITES of SATURN have their Distances measured in Semidiameters of his RING, whose Dimension in English Miles must therefore be first investigated. By the ROYAL ACADEMY OF SCI-ENCES at PARIS, the faid Ring if view'd at the Distance of the Earth from the Sun, would fubtend an Angle of 6. 41"=401". This differs confiderably from DR. Pound's measurement with a Micrometer in the Hugenian TELESCOPE of 123 feet, who made the true Proportion of the Diameter of Saturn to the greatest Diameter of the Ring to be that of 3 to 7. And fince Saturn's Diameter at the Distance of the Earth was 152,64 (4141) if we fay as 3:7:: 152,64,: 356,16, this last Number will be the Diameter of the Ring=5,56, which is 45" less than the Academicians make it. We may therefore suppose it not wide from Truth if we flate the Apparent Diameter of the Ring at 6'. 20"; and the Semidiameter

3'. 10"=190." compared with that of the Earth 8,78: will give this Analogy, As 8,78: 190":: 3975,7: 86035 English Miles in the Semidiameter of the Ring: And if the Width of the Ring and the Distance of its internal Edge from Saturn's Surface be as they are usually drawn'; they will have the Ratio of about 27500 to 30492 Miles.

4168. The Numbers in the 2d Column of the following Table being multiplied severally into 86035, will produce the Numbers in the 3d Column of English Miles.

Satellites.	Distances in Semid. of the Ring.		Time of Revolu- tion.	elocities Minute,
First Second Third Fourth	2,686	180415,2 231090 322803 748332,5	1. 21. 18. 27 2. 17. 41. 22 4. 12. 25. 12	Miles. 419,64 336,1 281,7

416g. Also the VELOCITES of the Satellites are deduced from their distances in Miles, and their mean motions in their Orbits per Minute, as stated by Dr. HAL-LEY thus. (1) 480"-(2) 300"-(3) 180"-(4) 56"-(5.) 11". And from thence it is manifest, there is but one, viz. the First Satellite, whose Velocity exceeds that of the Primary (see Table at 4154;) and therefore only that Satellite of the 5, can appear retrograde, as viewed from the Sun.

4170. Having thus finish'd our SURVEY of the PLA-NETARY SYSTEM, we may proceed to that of the Co-METS; which though it vastly exceeds the other, is much less known to us; indeed, we have not yet the Number of the Comets known; and of those that are known

known (though near 50) there is only one whose Orbit and Period are determined, which we owe to the Sagacity of the late great Dr. HALLEY; and have represented in Fig. 4. in the due Proportion of every Part, as in the Heavens.

N. B. In this Figure the dotted Arch ach is Part of the Orbit of Saturn.

4171. By the most diligent and accurate Observations on 3 different Periods, between its 4 several Appearances in 1534, 1607, 1682, and 1758; he collected the following ELEMENTS of its MOTION (1.) That its PE-RIODICAL TIME is 752 Years at a mean (2.) And that therefore the longer Semiaxis of its Elliptic Orbit, viz. EP=1786350 of fuch Parts as the Earth's Mean Diffance from the Sun is 100000. (3.) That the Perihelion Distance PS=58250. (4.) Whence the Eccentricity SE=1728100. (5.) And the leffer Semiaxis EC= 452460. (6.) The INCLINATION of its ORBIT to the Plane of the Ecliptic is 17°. 42'. (7.) It's Afcending None in 8. 20°. 48'. (8) The equated TIME of its PERIHELION Sepr. 4D. 21H. 22'. (9.) And laftly, that its Mean diurnal Motion at C or D was 47" of a Degree.

4172. These proportional Distances in Cometary OR-BITS are turned into English MILES by the usual Analogies; thus for the Comet's Mean Distance, or longer Semiaxis PE, you say, as 100000: 1786350:: 93399590: 1668440100; and twice that Number is 3336880200 English Miles in the Transverse Axis AP of the Orbit; viz. almost 3337 Millions of Miles.

4173. Again fay, As 100000: 93399590:: 58250: 54405263 Miles, for the Perihelion Diffance SP; which is little more than half the Earth's Mean Diffance from the Sun. Hence the Eccentricity SE=1614034837 Miles.

and its DIMENSIONS reduced to English MILES. 17

4174. Then for the leffer Semiaxis CE, fay, as 100000: 93399590:: 452460: 422595880; and there-

fore CD=84519176 English Miles.

4175. In order to determine the VELOCITY of any COMET'S MOTION at the Perihelion P, it has been shewn. (1.) That the Velocity of the Comet in Perihelio is not sensibly different from that in the Vertex of a Parabola of the same focal Distance, if the Eccentricity be very great (see Inst. 1244.) (2.) That the Velocity of a Planet is inversely as the Square Root of its Distance from the Sun (1171.) (3.) That the Velocity of a Comet in its Perihelion P, is to that of a Planet in a Circle at the same Distance PS, as $\sqrt{2}$ to 1. (1205, &c.)

4176. Therefore let APB be a Parabola (Fig. 5.); PDC the Orbit of a Planet at the same socal Distance PS; and T a Portion of the Earth's Orbit. Also v, U, and V, denote the Velocities of the Earth at T, and of the Planet and Comet at P. And let ST=D, and SP=d. Then we have $v:U::\sqrt{d}:\sqrt{D}$. Also, $V:U::\sqrt{2}:1$. These two Analogies give this third, viz. $v:V::\sqrt{d}:\sqrt{2D}$.

4177. Now in regard to the Comet under Confideration, we have d=58250, and D=100000, and fo 2D=200000; therefore, $v:V::\sqrt{58250}:\sqrt{200000}::241,35:447,21$. But the Velocity of the Earth per Minute is 1117,1 Miles; therefore fay, as 241,35:447,21::1117,1:2070=Velocity of the Comet in Perihelia P, viz. 2000 and 70 Miles per Minute.

4178. Because the Velocity in every Part of the Ellipsis is inversely as a Perpendicular from S to the Tangent in that Part (by 1213.) Therefore say, as CE: SP::452460:58250::2070:266,48=mean Velocity of the Comet at C or D, viz. 266 ½ Miles per Minute. (Fig. 4.)

D

4179. Lastly; the Velocity of this Comet per Minute, in it's most remote or Aphelion Point A, is had by saying, as AS: SP:: 3514450: 58250:: 2070: 34,308 or 34.30 Miles per Minute. Hence it appears that this Comet describes the same Space in one Minute at the Perihelion P, as it passes through in one Hour at its Aphelion A.

4180. Dr. HALLEY observes, there is not the same Certainty of the Return of the wonderful Comet of 1680, (which gave Sir I. Newton the greatest Trial of Skill), as in that we have just now dismissed; but still he thinks there is good Reason to expect it after a Period of no less than 575 Years; for he makes it very probable that it is the same Comet which appeared in 1106 mentioned in the Saxon Chronicle; and in 531 in the Reign of the Emperor Justinian; and in the Year 44 before Christ, mentioned by Augustus Cæsar himself as may be seen in Pliny's Nat. Hist. Book 2. Chap. 24.

4181. In confidence of this, the Doctor took the Pains to calculate a Table of its Motion, and from thence made a Computation of the Comet's Place for a given Time when it was observed, and the Agreement of all Phænomena between the Computation and Observation was beyond Expectation accurate. From this Period of 575 Years, he finds the greater Axis AP of the Ellipsis to be 13829570; the lesser Axis CD=184820; the Perihelion Distance PS=617,5; and the Aphelion Distance AS=13828952,5 of such Parts as the Earth's Distance contains 100000.

4182. These Distances are reduced to English Miles, as before, in the other Comet.—Thus 100000: 93399590:: 617,5: 576742,6=PS, the Perihelion Distance in Miles.—Then again, as 100000: 93399590:: 94210: 86310580=CE, the lesser Semiaxis.—And lastly, as 100000: 93399590:: 6914785: 6458548000=

PE, the greater Semiaxis; therefore the whole transverse Axis AP of this vast Ellipsis is 1291709600, that is, Twelve Thousand Nine Hundred and Seventeen Millions of English Miles !

4183. We have before found the Semidiameter of the Sun to be 362515 Miles (4154), if this be taken from this Comet's Perihelion Distance 576742,6 there will remain 214427,6 Miles, the Distance of the Comet from the Sun's Surface at P, but little more than a third Part of the Sun's Semidiameter. From the above Measures; too, it appears that the Length of this Ellipsis AP is to its Width CD, as 75 to 1, nearly. And lastly, that the Comet's greatest Distance from the Sun AS is to it's least Distance PS, as 22395 to 1. The LIGHT and HEAT at the Comet in it's Aphelion A, will be therefore 501540000 Times less than at P, that is, more than 500 Million of Times less.

4184. With respect to the Velocity of the Comet at its Perihelion P, that will be to the Velocity of the other Comet (4181) inversely as the Square Root of the Perihelion Distances; therefore say, $\sqrt{617}$: $\sqrt{58250}$:: 2070: 20104.707 Miles per Minute; so that the Perihelion Velocity of this Comet is near 10 Times that of the other.

4185. The Velocity in Perihelion P, is to that in the Aphelion A, as AS to PS; therefore fay, as 22395: 1::20104,707:0,89189=1580 Yards per Minute, 1760 Yards being a Mile.

4186. The Perihelion Distances of a great Number of Comets are known, but the Aphelion Distances only of these two; and by the Rule above (4177) they are all reduced to English Miles, as in the following Table.

Comets,	Periheli-	The fame in	
An. Dom.	on Dist.	English Miles	Perihelion.
1337	40666	3,7982307	D. H. June 2. 6. 35
1472	54273	50692000	Feb. 18. 22. 44
1531	56700	52957506	Aug. 24. 21. 27
1532	50910	47549702	Oct. 19. 22. 5
1556	46390	43328040	Apr. 21. 20. 10
1577	18342	17131306	Oct. 26. 18. 39
1580	59628	55705100	Nov. 28. 15. 3
1585	109358	102139800	Oct. 7. 19. 18
1590	57661	53855123	Feb. 8. 4. 10
1593	8911	8323100	July 18. 13. 48
1596	51293	47907498	Aug. 10. 20. 10
1607	58680	54806862	Oct. 26. 3. 44
1618	37975	35468518	Nov. 8. 12. 17
1652	84750	79156219	Nov. 12. 15. 34
1661	44851	4180640	Jan. 27. 0. 4
1664	102575 ¹	95805110	Dec. 4. 11. 53
1665	10649	9946121	Apr. 24. 5. 23
1672	69739	65136000	Mar. 1. 9. 0
1677	28059	26206325	May 6. 0. 43
1680	617 ¹	576742,6	Dec. 18. 0. 13
1682	58250	54405263	Sep. 14. 7. 44
1683	56020	52322450	July 13. 3. 5
1684	96015	89677600	June 8. 10. 24
1686	32500	30354850	Sep. 16. 14. 37
1698	69129	64566160	Oct. 18. 16. 52
1699	74400	69488830	Jan. 13. 8. 32
1702	64590	60326777	Mar. 13. 14. 22
1706	42581	39770683	Jan. 30. 4. 32
1707	85974	80299540	Dec. 11. 23. 39
1718	102655	95879345	Jan. 30. 4. 32
1723	99865	93273664	Sep. 27. 16. 20
1729	426141	398012740	June 25. 11. 6
1737	22283	20811582	Jan. 30. 8. 30
1739	67358	62911929	June 17. 10. 9
1742	76586	71514483	Feb. 8. 4. 48
1743	83501	77989290	Jan. 10. 20. 35
1744	22249½	20781172	Mar. 1. 8. 13
1747	219851	205339760	Mar. 3. 7. 20
1748	84067	78517973	Apr. 28. 19. 35
1762	101240	94557800	May 28. 15. 27
1770	62757½	58615240	Aug. 8. 19. 26
	90576	84607000	Apr. 18. 22. 14

A187. We have now finished our Survey of the Solar System; but before we desift, it may be worth while to shew at what an immense Distance the Systems of Planetary Worlds are placed from each other. In order hereto, it must be considered, that no Star of the First Magnitude (not Sirius itself) is near enough to have a Parallax of one Second, as we are fully assured by the most accurate Experiments of the late Dr. Bradley. Then let it be supposed that a Star were brought so near as to have a Parallax of 1". In this Case, the Diameter of the Earth's Orbit being at a Mean, 186799180 Miles, we say, as the Tangent of 1" - 4.685575 To Radius - 10.

So is the Diameter of the Orbit 186799180 8,271375

To the Distance of the Star 3853000000000 13,585800

That is, a Star near enough to have a Parallax of 1", would be at least 38530 Thousand Millions of Miles distant from our Sun; which is 412530, or 4125 Thousand Times the Distance of the Sun from us. Such a Star would also be 29840 Times farther from the Sun than the Comet of 1680 at its Aphelion Distance (see 4182.)

4188. By the above-mentioned Observations of Dr BRADLEY, it was found that Light took up 8'. 13" of Time in passing over the Semidiameter of the Earth's Orbit; therefore we may say, as 93399590: 8,217: 238530000000000: 6,445 Years; that is, Light would be 6½ Years nearly in coming from the Star to us, though its Velocity be more than 11 Million of Miles per Minute; for 8,217)93399590(=11367000.

4189. From this Demonstration of the immense Diftance of the nearest fixed Star, we have the two following Consectaries, viz. (1.) That a Body visible at the Distance of a Star, must be of a prodigious Magnitude; and (2.) That however large it were, it could not be

Predicaments are all that enter the Definition of a SUN; therefore every fixed Star is a Sun; fimilar to our own, no doubt; and destined for the same Purpose, viz. To be the Primum Mobile of a SYSTEM of Planetary WORLDS. And where is the absurdity to suppose the Universal Space repleat with an Infinity of Mundane Systems? I question if the infinite Power and Wisdom of the Deity be more conspicuous and amazing in the Construction of a Solar System than in the Formation and Organization of an Animalcule, a Million of times less than a Mite! Yet these abound, by Myriads, in every Puddle of Water.

APPENDIX.

The Description of two New Pieces of Mechanism for producing the proper Motions of the Sun and Venus, in the Partial Transit for London, on the large Celestial Landscape; and the Universal Transit, as shewn by the Grand Tellurian.

The Transit of a Planet over the Sun's Disk being the most curious Phænomenon which mortal eyes can behold, especially that of Venus, on account both of its Rarity and extreme Usefulness, I have endeavoured by a various Apparatus of Instruments, to render its Nature and wonderful Phases as intelligible to every Person's Understanding as I possibly could. Amongst these is a piece of Mechanism as new as the Phæne-

Phænomenon it exhibits to the Spectator's view, viz. An Artificial TRANSIT of VENUS, in which the Motion of the Sun, and of the Planet on its Disk, is represented just as it appeared to us at London, from the Ingress to Sun-set, in a Large CELESTIAL LANDSCAPE, 7½ Feet long, and 5 Feet high.

4191. This Landscape takes in that Part of the Heavens which the Sun passed through from VII. o'Clock to his Setting on the Evening of June 3, 1769, as it was beheld from Mr. Johnson's Great Room at Islington, in which the Appearance of the Sun, and the Transit of the Planet on his Disk was exhibited in a Manner very different from what was practised in every other Place. The lower Parts are a View of the Grounds between Islington and the Visible Horizon, terminated by the Hills towards Hampstead on one Part, and Whitsiela's Tabernacle on the other, as represented, in Small, in the Copper Plate.

4192. The Diagonal dotted Line is the Sun's Path or Ecliptic divided into Minutes by the Dotts, begining at VII. and ending at 5' after VIII, when the Sun went down on that memorable Day. The Sun is represented in 5 Positions; in the First it is supposed to appear from under a Cloud at VII o'clock. In the Second the Planet just touches the upper or vertical Point of the Sun's Disk at about 8\frac{3}{4} Minutes after VII. In the Third, the Planet is wholly enter'd, and touches the Disk internally, at about 27\frac{1}{2} after VII. In the Fourth the Planet is shewn soon after the internal Contact; and in the Fish and last Position the Sun is shewn setting, with the Planet just as far on the Disk as it was then seen, by the Spectators, in the Horizon.

4193. The Motion of the Sun is produced by Mechanism like that of a Clock, confisting of a strong Spring and Fusee, with a proper Train of Wheel-Work regulated

regulated by a Fly; by which Means a filk Cord is drawn in the Direction of the Diagonal or Ecliptic Line behind the Frame; to which Line a piece of Wood is fixed, and is made to move with its central Part in a Groove or Slic cutthrough the Diagonal of the Landscape from Corner to Corner.

4194. The Motion of VENUS upon the Solar Difk is effected by a Piece of Mechanism whose Rationale and Construction is to be understood as follows. The Apparent Motion of the Sun's Center at the Time of his Setting on June the 3d, was at the Rate of 696'. 40" per Hour; for let B (Fig. 6.) be the Center of the Sun when his limb first touches the Horizon at A; and C, his Center in the Horizon. Then by a Theorem (in the Mariner's Mirror, Page 50.) You find the Time Spent in the Sun's Setting, viz. 4'. 16"; therefore half that Time is spent in passing from B to C, viz. 2'. 8". But the Length of BC is known from the Given Semidiameter of the Sun AB=15'. 45" and the angle ACB= 30°. 29'. that is, BC=24'. 46"; therefore fay, As 2'. 8"; 24'. 46" :: 60' : 696,6. The Motion of Venus was just 4' per Hour; therefore fay, as 4':696', 6::1; 174, 15.

195. Hence it follows that fince the Motion of Venus is to that of the Sun as 1 to 174, then a Train of Wheel-work will be easily found to produce that Difference of Motion (3180, &c.) and consequently to carry the Planet over the Solar Disk in the Manner required; and that is such as is drawn in the Figure, viz. A Wheel of 58 and Pinion of 10; which drives another of 48 and Pinion of 8; and this moves a third Wheel of 30, whose Pinion of 6 plays in the Teeth of a long fine slip of Brass GF placed in the direction of the Planets Path over the Disk. Thus

$$\frac{10}{58} \times \frac{8}{48} \times \frac{6}{30} = \frac{1}{174}$$
, the Train required.

4196. This first Wheel marked 58 in the Figure has not Teeth, but a Groove cut its Periphery, and is with its Pinion fixed to a brass Pipe or Socket which is moveable on a Pin or Axis, fixed in the Piece of Wood before mentioned (4193) by a String going round it, and properly strained and fixed to the back part of the Frame

4197. This Socket with its Pinion of 10 comes through the Landscape, and moves in a long Slit cut through the Whole in the Direction of the Sun's Path, or dotted Line in the Landscape; the other Part of the Mechanism is placed upon a Plate on the fare side of the said Landscape, and over the Whole is fixed a circular silver'd Plate of 3 Inches Diameter to represent the Sun. The slip GF under the Solar Plate carries a fine black Wire FH to an Inch distance beyond it, and is there bent back to carry the small black circular Piece F, or Planet, over the Face of the Sun GFD. And thus the Artificial Transit in all Respects corresponds to the Natural One, and gives great Satisfaction when exhibited in the Course of my Lectures.

Transit is only partial, for the Phases at London; but there is another less complicated Piece of Mechanism which exhibits an artificial view of the Transit universally, viz. during it's whole Appearance, in every Phase, and to every Part of the World, where it was, in any degree visible, and the Time of day from Sun rising to Sun-set. This is done by means of the Grand Tellurian by Clock-Work, as follows.

4199. The Dial Plate of the Tellurian is divided into 12 Hours, and every Hour into 60 Minutes by an Horologic Vernier; the Circumference of this Plate is divided into 12 times 16, or 192 Teeth; to these a small Wheel is adapted with 36 Teeth, for an Exhibition of Solar Eclipses. In this Wheel of 36 another plays with

30 teeth, upon which is fixed a circular Piece of Box with a Groove in it's Circumference to wind on a Silken String, when the Globe is in Motion.

4200. At the other End of the Table is placed on a proper Stand, a square Frame, with a circular silvered Plate, 12 Inches diameter, (whose Center is just as high as that of the Tellurian) to represent the Disk of the Sun, as it was actually shewn in the darken'd Room on a Screen in a similar Position, at the Time of the Transit.

4201. On this filver'd Solar Disk are drawn three Lines, one for the Ecliptic through the Center, horizontally; another for the Path of Venus as view'd from the Earth's Center; and a third from the Center, perpendicular to the said Transit Line. By the Sides of the Frame are disposed certain Pullies conducting the aforesaid silk String in such Manner that it always coincides with the Transit Line. Lastly, upon a proper Part of this String is a round black Patch fixed, about 30 of an Inch wide, to reprehend the sable Planet on the Solar Surface.

4202. The Duration of the Central Transit being 1.
6. 20'. \(\frac{1}{2}\) (4068) and the length of the Chord, or Path of Venus, on the filver'd Plate, just 9,3 Inches; if we fay as 16: 1:: 30'. 1, 87 = the Time in which the Wheel of 30 makes one Revolution; therefore fay, 6,33: 9,3: 1,87: 2,75 Inches described in the Path in the time of 1.
1, 87. Then, lastly, say As, 3,14159: 1:: 2,75: 0,875 = the Diameter of the Pulley above mentioned, to give the same Motion to the Venereal Patch on the filvered Plate as the Planet itself had upon the Disk of the Sun.

4203. Such are the Principles of this Machinery; and now having rectified the Tellurian for the 3d Day

of June at Noon, and the Planetary Patch duly adjusted on the black Ground of the Frame, but not yet connected with the Pulley; the Machine is put into Motion. which exhibits a Natural View of all the Parts of the Earth in Motion from Noon till VII in the Evening, when the Motion is flop'd, to connect the Spring with the Pulley; and then it is put into Motion again; And just as the Index points to o Minutes after VII, the Patch is feen to touch the Limb of the Sun externally; in about 18 more, it appears wholly upon the filver'd Disk; from whence it proceds to the western Limb of the Sun, where it goes off, and disappears, at about 29 minutes after I in the Morning.

4204. During the time of the Transit, the Surface of the Tellurian in the luminous Hemisphere, presents to your View, all the Inhabitantst of the Earth, to whom every respective Phase of the Transit is visible, and at what Moment of Time; for all who enter it, fee the Phænomena at Sun-rifing; all who are going out of it into the dark Hemisphere, view them at Sun-set; and to all that are upon the Solar Meridian, they appear at Mid-day. Also all those to whom the Whole will be visible; others who can fee nothing at all of it; and, laftly; those who fee but Part of the Transit, more or less.

SCHOLIUM.

Such are the Methods of exhibiting to Posterity the most extraordinary, sublime, and useful Phanomenon of Nature, which they might very naturally have envied us the Enjoyment of, were it not that by these Species of Mechanism they have it in their power to reiterate, as it were, the Transit of this Planet whenever they please, in all the Circumstances it could ever be attended with, excepting not even the Actual Observation of it; for this,

in the way I took to exhibit the Phænomenon, was in all respects similar to that by the Tellurian, viz. The Room was darken'd and a Hole in the Window-Shutter gave a cylindric Beam of the Sun's Rays into the Tube of a Reflecting Telescope, whose Metals form'd the Image of the Sun and Planet upon a Screen placed at a proper Distance from it, in so large, distinct and perspicuous a Manner, that more than 50 People could eafily fee it at once, and observe each particular Phase as it arose. For the Image of the Sun was about 15 Inches Diameter, and Venus, of Course, near half an Inch, in Form of a black Patch, perfectly round; and free from any of those monstrous Phases occasioned by Parallax and Refraction thro' Glaffes in Telescopes, where the Image of the Sun could be little more than \frac{1}{2} or \frac{3}{4} of an Inch, and that of Venus not more than 10 of an Inch, which Images must be view'd thro' deep colorific Lenses, and then no wonder if Visions and Phantoms arose of indented and lacerated Limbs of the Sun; piriform, Bulbons, and bottle-belly'd Planets, furrounded with Circles of Light, and luminous Zones, which must be construed into an Atmosphere about Venus, 500 times higher than that about our Earth. -- But these unnatural Appearances could have no Place in my way of exhibiting the Transit; the Planet touched the Sun upon the Screen just as the Patch touches the Silver'd Plate at the external contact; and the moment after the internal, the Planet became a round black fpot, perfectly well defined; For the Truth of this I can appeal to Hundreds now living, who faw these Phases at the Egress of the Planet in the first Tranfit in 1761, and at the Ingress in the last of 1769; When the moment of the external Contact was, by a mean of

several Stop-Watches, 7.8'. 52".

